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A note on spin-s duality

Nicolas Boulanger,* Sandrine Cnockaert† and Marc Henneaux‡

Physique Théorique et Mathématique, Université Libre de Bruxelles
C.P. 231, B-1050 Bruxelles, Belgium
E-mail: nboulang@ulb.ac.be, scnockae@ulb.ac.be, henneaux@ulb.ac.be

Abstract: Duality is investigated for higher spin ($s \geq 2$), free, massless, bosonic gauge fields. We show how the dual formulations can be derived from a common "parent", first-order action. This goes beyond most of the previous treatments where higher-spin duality was investigated at the level of the equations of motion only. In $D = 4$ spacetime dimensions, the dual theories turn out to be described by the same Pauli-Fierz ($s = 2$) or Fronsdal ($s \geq 3$) action (as it is the case for spin 1). In the particular $s = 2$ $D = 5$ case, the Pauli-Fierz action and the Curtright action are shown to be related through duality. A crucial ingredient of the analysis is given by the first-order, gauge-like, reformulation of higher spin theories due to Vasiliev.

Keywords: Duality in Gauge Field Theories, Gauge Symmetry.

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*"Chercheur F.R.I.A.", Belgium.
†Aspirante du Fonds National de la Recherche Scientifique (Belgium).
‡Also at Centro de Estudios Científicos, Casilla 1469, Valdivia, Chile.

Contents

1. Introduction

Duality for higher spin massless gauge fields has been the focus of a great interest recently [1]–[6]. In most of this recent work, duality is studied at the level of the equations of motion only (a notable exception being [1, 6], which deals with the spin-2 case in 4 spacetime dimensions). One may wonder whether a stronger form of duality exists, for all spins and in all spacetime dimensions, which would relate the corresponding actions. A familiar example in which duality goes beyond mere on-shell equivalence is given by a set of a free p-form gauge field and a free $(D - p - 2)$-form gauge field in D spacetime dimensions. The easiest way to establish the equivalence of the two theories in that case is to start from a first order "mother" action involving simultaneously the p-form gauge field

$A_{\mu_1 \cdots \mu_p}$ and the field strength $H_{\mu_1 \cdots \mu_{D-p-1}}$ of the $(D-p-2)$-form $B_{\mu_1 \cdots \mu_{D-p-2}}$ treated as independent variables

$$S[A,H] \sim \int dA \wedge H - \frac{1}{2} H \wedge {}^* H\,. \tag{1.1}$$

The field H is an auxiliary field that can be eliminated through its own equation of motion, which reads $H = {}^*dA$. Inserting this relation in the action (1.1) yields the familiar second-order Maxwell action $\sim \int dA \wedge {}^*dA$ for A. Conversely, one may view A as a Lagrange multiplier for the constraint $dH = 0$, which implies $H = dB$. Solving for the constraint inside (1.1) yields the familiar second-order action $\sim \int dB \wedge {}^*dB$ for B.

Following Fradkin and Tseytlin [7], we shall reserve the terminology "dual theories" for theories that can be related through a "parent action", referring to "pseudo-duality" for situations when there is only on-shell equivalence. The parent action may not be unique. In the above example, there is another, "father" action in which the roles of A and B are interchanged (B and F are the independent variables, with $S \sim \int dB \wedge F - \frac{1}{2} F \wedge {}^* F$ and $F = dA$ on-shell). That the action of dual theories can be related through the above transformations is important for establishing equivalence of the (local) ultraviolet quantum properties of the theories, since these transformations can formally be implemented in the path integral [7].

Recently, dual formulations of massless spin-2 fields have attracted interest in connection with their possible role in uncovering the hidden symmetries of gravitational theories [8]–[14]. In these formulations, the massless spin-2 field is described by a tensor gauge field with mixed Young symmetry type. The corresponding Young diagram has two columns, one with $D-3$ boxes and the other with one box. The action and gauge symmetries of these dual gravitational formulations have been given in the free case by Curtright [15]. However, the connection with the more familiar Pauli-Fierz formulation [16] was not clear and direct attempt to prove equivalence met problems with trace conditions on some fields. The difficulty that makes the spin-1 treatment not straightforwardly generalizable is that the higher-spin ($s \geq 2$) gauge lagrangians are not expressed in terms of strictly gauge-invariant objects, so that gauge invariance is a more subtle guide. One of the purposes of this note is to show explicitly that the Curtright action and the Pauli-Fierz action both come from the same parent action and are thus dual in the Fradkin-Tseytlin sense. The analysis is carried out in any number of spacetime dimensions and has the useful property, in the self-dual dimension four, that both the original and the dual formulations are described by the same Pauli-Fierz lagrangian and variables.

We then extend the analysis to higher spin gauge fields described by completely symmetric tensors. The lagrangians for these theories, leading to physical second-order equations, have been given long ago in [17]. We show that the spin-s theory, described in [17] by a totally symmetric tensor with s indices and subject to the double-tracelessness condition, is dual to a theory with a field of mixed symmetry type $(D-3, 1, 1, \ldots, 1)$ (one column with $D-3$ boxes, $s-1$ colums with one box), for which we give explicitly the lagrangian and gauge symmetries. This field is also subject to the double tracelessness condition on any pair of pairs of indices. A crucial tool in the analysis is given by the first-order reformulation of the Fronsdal action due to Vasiliev [18], which is in fact our starting point. We find again that in the self-dual dimension four, the original description and the dual description are the same.

2. Spin-2 duality

2.1 Parent actions

We consider the first-order action [10]

$$\mathcal{S}\left[e_{ab}, Y^{ab|}{}_{c}\right] = -2\int d^Dx\left[Y^{ab|c}\partial_{[a}e_{b]c} - \frac{1}{2}Y_{ab|c}Y^{ac|b} + \frac{1}{2(D-2)}Y_{ab|}{}^{b}Y^{ac|}{}_{c}\right] \tag{2.1}$$

where e_{ab} has both symmetric and antisymmetric parts and where $Y^{ab|}{}_{c} = -Y^{ba|}{}_{c}$ is a once-covariant, twice-contravariant mixed tensor. Neither e nor Y transform in irreducible representations of the general linear group since e_{ab} has no definite symmetry while $Y^{ab|}{}_{c}$ is subject to no trace condition. Latin indices run from 0 to $D-1$ and are lowered or raised with the flat metric, taken to be of "mostly plus" signature $(-,+,\ldots,+)$. The spacetime dimension D is ≥ 3. The factor 2 in front of (2.1) is inserted to follow the conventions of [18].

The action (2.1) differs from the standard first-order action for linearized gravity in which the vielbein e_{ab} and the spin connection $\omega_{ab|c}$ are treated as independent variables by a mere change of variables $\omega_{ab|c} \to Y^{ab|}{}_{c}$ such that the coefficient of the antisymmetrized derivative of the vielbein in the action is just $Y^{ab|}{}_{c}$, up to the inessential factor of -2. This change of variables reads

$$Y_{ab|c} = \omega_{c|a|b} + \eta_{ac}\omega^{i}{}_{|b|i} - \eta_{bc}\omega^{i}{}_{|a|i}\,; \qquad \omega_{a|b|c} = Y_{bc|a} + \frac{2}{D-2}\eta_{a[b}Y_{c]d|}{}^{d}\,.$$

It was considered (for full gravity) previously in [10].

By examining the equations of motion for $Y^{ab|}{}_{c}$, one sees that $Y^{ab|}{}_{c}$ is an auxiliary field that can be eliminated from the action. The resulting action is

$$\mathcal{S}[e_{ab}] = 4\int d^Dx\left[C_{ca|}{}^{a}C^{cb|}{}_{b} - \frac{1}{2}C_{ab|c}C^{ac|b} - \frac{1}{4}C_{ab|c}C^{ab|c}\right] \tag{2.2}$$

where $C_{ab|c} = \partial_{[a}e_{b]c}$. This action depends only on the symmetric part of e_{ab} (the lagrangian depends on the antisymmetric part of e_{ab} only through a total derivative) and is a rewriting of the linearised Einstein action of general relativity (Pauli-Fierz action).

From another point of view, e_{ab} can be considered in the action (2.1) as a Lagrange multiplier for the constraint $\partial_a Y^{ab|}{}_{c} = 0$. This constraint can be solved explicitely in terms of a new field $Y^{abe|}{}_{c} = Y^{[abe]|}{}_{c}$, $Y^{ab|}{}_{c} = \partial_e Y^{abe|}{}_{c}$. The action then becomes

$$\mathcal{S}[Y^{abe|}{}_{c}] = 2\int d^Dx\left[\frac{1}{2}Y_{ab|c}Y^{ac|b} - \frac{1}{2(D-2)}Y_{ab|}{}^{b}Y^{ac|}{}_{c}\right] \tag{2.3}$$

where $Y^{ab|c}$ must now be viewed as the dependent field $Y^{ab|c} = \partial_e Y^{abe|c}$. The field $Y^{abe|}{}_{c}$ can be decomposed into irreducible components: $Y^{abe|}{}_{c} = X^{abe|}{}_{c} + \delta^{[a}_{c}Z^{be]}$, with $X^{abc|}{}_{c} =$

0, $X^{abe|}{}_c = X^{[abe]|}{}_c$ and $Z^{be} = Z^{[be]}$. A direct but somewhat cumbersome computation shows that the resulting action depends only on the irreducible component $X^{abe|}{}_c$, i.e. it is invariant under arbitrary shifts of Z^{ab} (which appears in the lagrangian only through a total derivative). One can then introduce in $D \geq 4$ dimensions the field $T_{a_1 \cdots a_{D-3}|c} = \frac{1}{3!}\epsilon_{a_1 \cdots a_{D-3} efg} X^{efg|}{}_c$ with $T_{[a_1 \cdots a_{D-3}|c]} = 0$ because of the trace condition on $X^{efg|}{}_c$, and rewrite the action in terms of this field.[1] Explicitly, one finds the action given in [15, 19]:

$$\begin{aligned}
\mathcal{S}[T_{a_1 \cdots a_{D-3}|c}] = -\frac{1}{(D-3)!}\int d^D x \times \\
\times \Bigg[\partial^e T^{b_1 \ldots b_{D-3}|a} \partial_e T_{b_1 \ldots b_{D-3}|a} - \partial_e T^{b_1 \ldots b_{D-3}|e} \partial^f T_{b_1 \ldots b_{D-3}|f} - \\
- (D-3) \Big[-3 \partial_e T^{e b_2 \ldots b_{D-3}|a} \partial^f T_{f b_2 \ldots b_{D-3}|a} - \\
- 2 T_g{}^{b_2 \ldots b_{D-3}|g} \partial^{ef} T_{e b_2 \ldots b_{D-3}|f} - \\
- \partial^e T_g{}^{b_2 \ldots b_{D-3}|g} \partial_e T^f{}_{b_2 \ldots b_{D-3}|f} + \\
+ (D-4) \partial_e T_g{}^{e b_3 \ldots b_{D-3}|g} \partial^h T^f{}_{h b_3 \ldots b_{D-3}|f} \Big] \Bigg] .
\end{aligned} \tag{2.4}$$

By construction, this dual action is equivalent to the initial Pauli-Fierz action for linearised general relativity. We shall compare it in the next subsections to the Pauli-Fierz ($D = 4$) and Curtright ($D = 5$) actions.

One can notice that the equivalence between the actions (2.2) and (2.3) can also be proved using the following parent action:

$$\mathcal{S}[C_{ab|c}, Y_{abc|d}] = 4 \int d^D x \left[-\frac{1}{2} C_{ab|c} \partial_d Y^{dab|c} + C_{ca|}{}^a C^{cb|}{}_b - \frac{1}{2} C_{ab|c} C^{ac|b} - \frac{1}{4} C_{ab|c} C^{ab|c} \right], \tag{2.5}$$

where $C_{ab|c} = C_{[ab]|c}$ and $Y_{abc|d} = Y_{[abc]|d}$. The field $Y_{abc|d}$ is then a Lagrange multiplier for the constraint $\partial_{[a} C_{bc]|d} = 0$, this constraint implies $C_{ab|c} = \partial_{[a} e_{b]c}$ and, eliminating it, one finds that the action (2.5) becomes the action (2.2). On the other hand, $C_{ab|c}$ is an auxiliary field and can be eliminated from the action (2.5) using its equation of motion, the resulting action is then the action (2.3).

2.2 Gauge symmetries

The gauge invariances of the action (2.2) are known: $\delta e_{ab} = \partial_a \xi_b + \partial_b \xi_a + \omega_{ab}$, where $\omega_{ab} = \omega_{[ab]}$. These transformations can be extended to the auxiliary fields (as it is always the case [21]) leading to the gauge invariances of the parent action (2.1):

$$\delta e_{ab} = \partial_a \xi_b + \partial_b \xi_a \,, \tag{2.6}$$

$$\delta Y^{ab|}{}_d = -\,6\,\partial_c\,\partial^{[a} \xi^b \delta_d^{c]} \tag{2.7}$$

[1] For $D = 3$, the field $X^{efg|}{}_c$ is identically zero and the dual lagrangian is thus $\mathcal{L} = 0$. The duality transformation relates the topological Pauli-Fierz lagrangian to the topological lagrangian $\mathcal{L} = 0$. We shall assume $D > 3$ from now on.

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and

$$\delta e_{ab} = \omega_{ab} \,, \tag{2.8}$$
$$\delta Y^{ab|}{}_{d} = 3\,\partial_c\,\omega^{[ab}\delta^{c]}_{d} \,. \tag{2.9}$$

Similarly, the corresponding invariances for the other parent action (2.5) are:

$$\delta C_{ab|c} = \partial_c \partial_{[a}\xi_{b]} \,, \tag{2.10}$$
$$\delta Y^{abc|}{}_{d} = -\,6\,\partial^{[a}\xi^{b}\delta^{c]}_{d} \tag{2.11}$$

and

$$\delta C_{ab|c} = \partial_{[a}\omega_{b]c} \,, \tag{2.12}$$
$$\delta Y^{abc|}{}_{d} = 3\,\omega^{[ab}\delta^{c]}_{d} \,. \tag{2.13}$$

These transformations affect only the irreducible component Z^{be} of $Y^{abe|}{}_{c}$. [Note that one can redefine the gauge parameter ω_{ab} in such a way that $\delta e_{ab} = \partial_a\xi_b + \omega_{ab}$. In that case, (2.6) and (2.7) become simply $\delta e_{ab} = \partial_a \xi_b$, $\delta Y^{ab|}{}_{d} = 0$.]

Given $Y^{ab|}{}_{c}$, the equation $Y^{ab|}{}_{c} = \partial_e Y^{abe|}{}_{c}$ does not entirely determine $Y^{abe|}{}_{c}$. Indeed $Y^{ab|}{}_{c}$ is invariant under the transformation

$$\delta Y^{abe|}{}_{c} = \partial_f \left(\phi^{abef|}{}_{c}\right) \tag{2.14}$$

of $Y^{abe|}{}_{c}$, with $\phi^{abef|}{}_{c} = \phi^{[abef]|}{}_{c}$. As the action (2.3) depends on $Y^{abe|}{}_{c}$ only through $Y^{ab|}{}_{c}$, it is also invariant under the gauge transformations (2.14) of the field $Y^{abe|c}$. In addition, it is invariant under arbitrary shifts of the irreducible component Z^{ab},

$$\delta Y^{abc|}{}_{d} = 3\,\omega^{[ab}\delta^{c]}_{d} \,. \tag{2.15}$$

The gauge invariances of the action (2.4) involving only $X^{abe|}{}_{c}$ (or equivalently, $T_{a_1\cdots a_{D-3}|c}$) are simply (2.14) projected on the irreducible component $X^{abe|}{}_{c}$ (or $T_{a_1\cdots a_{D-3}|c}$).

It is of interest to note that it is the same ω-symmetry that removes the antisymmetric component of the tetrad in the action (2.2) (yielding the Pauli-Fierz action for $e_{(ab)}$) and the trace Z^{ab} of the field $Y^{abe|}{}_{c}$ (yielding the action (2.4) for $T_{a_1\cdots a_{D-3}|c}$ (or $X^{abe|}{}_{c}$)). Because it is the same invariance that is at play, one cannot eliminate simultaneously both $e_{[ab]}$ and the trace of $Y^{ab|}{}_{c}$ in the parent actions, even though these fields can each be eliminated individually in their corresponding "children" actions (see [20] in this context).

2.3 $D = 4$: "Pauli-Fierz is dual to Pauli-Fierz"

In $D = 4$ spacetime dimensions, the tensor $T_{a_1\cdots a_{D-3}|c}$ has just two indices and is symmetric, $T_{ab} = T_{ba}$. A direct computation shows that the action (2.4) becomes then

$$\mathcal{S}[T_{ab}] = \int d^4x \left[\partial^a T^{bc}\partial_a T_{bc} - 2\partial_a T^{ab}\partial^c T_{cb} - 2T_a{}^a\partial^{bc}T_{bc} - \partial_a T_b{}^b\partial^a T_c{}^c\right] \tag{2.16}$$

which is the Pauli-Fierz action for the symmetric massless tensor T_{ab}. At the same time, the gauge parameters $\phi^{abef|}{}_c$ can be written as $\phi^{abef|}{}_c = \epsilon^{abef}\gamma_c$ and the gauge transformations reduce to $\delta T_{ab} \sim \partial_a\gamma_b + \partial_b\gamma_a$, as they should. Our dualization procedure possesses thus the distinct feature, in four spacetime dimensions, of mapping the Pauli-Fierz action on itself. Note that the electric (respectively, the magnetic) part of the (linearized) Weyl tensor of the original Pauli-Fierz field $h_{ab} \equiv e_{(ab)}$ is equal to the magnetic (respectively, minus the electric) part of the (linearized) Weyl tensor of the dual Pauli-Fierz T_{ab}, as expected for duality [22, 2].

An alternative, interesting, dualization procedure has been discussed in [6]. In that procedure, the dual theory is described by a different action, which has an additional antisymmetric field, denoted ω_{ab}. This field does enter non trivially the lagrangian through its divergence $\partial^a\omega_{ab}$.[2]

2.4 $D = 5$: "Pauli-Fierz is dual to Curtright"

In $D = 5$ spacetime dimensions, the dual field is $T_{ab|c} = \frac{1}{3!}\varepsilon_{abefg}X^{efg|}{}_c$, and has the symmetries $T_{ab|c} = T_{[ab]|c}$ and $T_{[ab|c]} = 0$. The action found by substituting this field into (2.3) reads

$$\mathcal{S}[T_{ab|c}] = \frac{1}{2}\int d^5x \Big[\partial^a T^{bc|d}\partial_a T_{bc|d} - 2\partial_a T^{ab|c}\partial^d T_{db|c} - \partial_a T^{bc|a}\partial^d T_{bc|d} - \\ - 4T_a{}^{b|a}\partial^{cd}T_{cb|d} - 2\partial_a T_b{}^{c|b}\partial^a T^d{}_{c|d} - 2\partial_a T_b{}^{a|b}\partial^c T^d{}_{c|d}\Big] . \tag{2.17}$$

It is the action given by Curtright in [15] for such an "exotic" field.

The gauge symmetries also match, as can be seen by redefining the gauge parameters as $\psi_{gc} = -\frac{1}{4!}\epsilon_{abefg}\phi^{abef|}{}_c$. The gauge transformations become

$$\delta T_{ab|c} = -\frac{1}{2}(\partial_a\psi_{bc} - \partial_b\psi_{ac}) = \partial_{[a}S_{b]c} + \frac{1}{3}\left[\partial_a A_{bc} + \partial_b A_{ca} - 2\partial_c A_{bc}\right], \tag{2.18}$$

where $\psi_{ab} = S_{ab} + A_{ab}, S_{ab} = S_{ba}, A_{ab} = -A_{ba}$. These are exactly the gauge transformations of [15].

It was known from [2] that the equations of motion for a Pauli-Fierz field were equivalent to the equations of motion for a Curtright field, i.e., that the two theories were "pseudo-dual". We have established here that they are, in fact, dual. The duality transformation considered here contains the duality transformation on the curvatures considered in [2]. Indeed, when the equations of motion hold, one has $R_{\mu\nu\alpha\beta}[h] \propto \varepsilon_{\mu\nu\rho\sigma\tau}R^{\rho\sigma\tau}{}_{\alpha\beta}[T]$ where $R_{\mu\nu\alpha\beta}[h]$ (respectively $R_{\rho\sigma\tau\alpha\beta}[T]$) is the linearized curvature of $h_{ab} \equiv e_{(ab)}$ (respectively, $T_{ab|c}$).

[2]In the lagrangian (27) of [6], one can actually dualize the field ω_{ab} to a scalar Φ (i.e., (i) replace $\partial^a\omega_{ab}$ by a vector k_b in the action; (ii) force $k_b = \partial^a\omega_{ab}$ through a Lagrange multiplier term $\Phi\partial^a k_a$ where Φ is the Lagrange multiplier; and (iii) eliminate the auxiliary field k_a through its equations of motion). A redefinition of the symmetric field $\tilde{h}_{ab}$ of [6] by a term $\sim \eta_{ab}\Phi$ enables one to absorb the scalar Φ, yielding the Pauli-Fierz action for the redefined symmetric field.

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3. Vasiliev description of higher spin fields

In the discussion of duality for spin-two gauge fields, a crucial role is played by the first-order action (2.1), in which both the (linearized) vielbein and the (linearized) spin-connection (or, rather, a linear combination of it) are treated as independent variables. This first-order action is indeed one of the possible parent actions. In order to extend the analysis to higher-spin massless gauge fields, we need a similar description of higher-spin theories. Such a first-order description has been given in [18]. In this section, we briefly review this formulation, alternative to the more familiar second-order approach of [17]. We assume $s > 1$ and $D > 3$.

3.1 Generalized vielbein and spin connection

The set of bosonic fields introduced in [18] consists of a generalized vielbein $e_{\mu|a_1\ldots a_{s-1}}$ and a generalized spin connection $\omega_{\mu|b|a_1\ldots a_{s-1}}$. The vielbein is completely symmetric and traceless in its last $s-1$ indices. The spin-connection is not only completely symmetric and traceless in its last $s-1$ indices but also traceless between its second and one of its last $s-1$ indices. Moreover, complete symmetrization in all its indices but the first gives zero. Thus, one has

$$\begin{aligned} e_{\mu|a_1\ldots a_{s-1}} &= e_{\mu|(a_1\ldots a_{s-1})}\,, & e_{\mu|}{}^{b}{}_{b\ldots a_{s-1}} &= 0\,,\\ \omega_{\mu|b|a_1\ldots a_{s-1}} &= \omega_{\mu|b|(a_1\ldots a_{s-1})}\,, & \omega_{\mu|(b|a_1\ldots a_{s-1})} &= 0\,,\\ \omega_{\mu|b|}{}^{c}{}_{c\ldots a_{s-1}} &= 0\,, & \omega_{\mu|}{}^{b}{}_{|b\ldots a_{s-1}} &= 0\,. \end{aligned} \tag{3.1}$$

The first index of both the vielbein and the spin-connection may be seen as a spacetime form-index, while all the others are regarded as internal indices. As we work at the linearized level, no distinction will be made between both kinds of indices and they will both be labelled either by Greek or by Latin letters, running from $0, 1, \ldots, D-1$.

The action was originally written in [18] in four dimensions as

$$\mathcal{S}^s[e,\omega] = \int d^4x\, \varepsilon^{\mu\nu\rho\sigma}\, \varepsilon_{abc\sigma}\, \omega_{\rho|}{}^{b|ai_1\ldots i_{s-2}}\left(\partial_\mu e_{\nu|i_1\ldots i_{s-2}}{}^{c} - 1/2\omega_{\mu|\nu|i_1\ldots i_{s-2}}{}^{c}\right). \tag{3.2}$$

By expanding out the product of the two ϵ-symbols, one can rewrite it in a form valid in any number of spacetime dimensions,

$$\begin{aligned} \mathcal{S}^s[e,\omega] = -2\int d^Dx \Bigg[&\left(B_{a_1[\nu|\mu]a_2\ldots a_{s-1}} - \frac{1}{2(s-1)}B_{\nu\mu|a_1\ldots a_{s-1}}\right) K^{\mu\nu|a_1\ldots a_{s-1}} + \\ &+ \left(2B^{\rho}{}_{\mu|a_2\ldots a_{s-1}\rho} + (s-2)B^{\rho}{}_{a_2|a_3\ldots a_{s-1}\mu\rho}\right) K^{\mu\nu|a_2\ldots a_{s-1}}{}_{\nu}\Bigg] \end{aligned} \tag{3.3}$$

where

$$B_{\mu b|a_1\ldots a_{s-1}} \equiv 2\omega_{[\mu|b]|a_1\ldots a_{s-1}} \tag{3.4}$$

and where

$$K^{\mu\nu|a_1\ldots a_{s-1}} = \partial^{[\mu}e^{\nu]|a_1\ldots a_{s-1}} - \frac{1}{4}B^{\mu\nu|a_1\ldots a_{s-1}}\,. \tag{3.5}$$

The field $B_{\mu b|a_1\ldots a_{s-1}}$ is antisymmetric in the first two indices, symmetric in the last $s-1$ internal indices and traceless in the internal indices,

$$B_{\mu b|a_1\ldots a_{s-1}} = B_{[\mu b]|a_1\ldots a_{s-1}}\,,\ B_{\mu b|a_1\ldots a_{s-1}} = B_{\mu b|(a_1\ldots a_{s-1})}\,,\ B_{\mu b|a_1\ldots a_{s-2}}{}^{a_{s-2}} = 0 \qquad (3.6)$$

but is otherwise arbitrary: given B subject to these conditions, one can always find an ω so that (3.4) holds [18].

The invariances of the action (3.2) are [18]

$$\delta e_{\mu|a_1\ldots a_{s-1}} = \partial_\mu \xi_{a_1\ldots a_{s-1}} + \alpha_{\mu|a_1\ldots a_{s-1}}\,, \qquad (3.7)$$
$$\delta \omega_{\mu|b|a_1\ldots a_{s-1}} = \partial_\mu \alpha_{b|a_1\ldots a_{s-1}} + \Sigma_{\mu|b|a_1\ldots a_{s-1}}\,, \qquad (3.8)$$

where the parameters $\alpha_{\mu|a_1\ldots a_{s-1}}$ and $\Sigma_{\mu|b|a_1\ldots a_{s-1}}$ possess the following algebraic properties

$$\begin{aligned}
\alpha_{\nu|(a_1\ldots a_{s-1})} &= \alpha_{\nu|a_1\ldots a_{s-1}}\,, \quad \alpha_{(\nu|a_1\ldots a_{s-1})} = 0\,, \quad \alpha^{\nu}{}_{|\nu a_2\ldots a_{s-1}} = 0\,, \quad \alpha_{\nu|a_1\ldots a_{s-3}b}{}^{b} = 0\,,\\
\Sigma_{\mu|b|a_1\ldots a_{s-1}} &= \Sigma_{(\mu|b)|a_1\ldots a_{s-1}} = \Sigma_{\mu|b|(a_1\ldots a_{s-1})}\,, \quad \Sigma_{\mu|(b|a_1\ldots a_{s-1})} = 0\,,\\
\Sigma^{b}{}_{|b|a_1\ldots a_{s-1}} &= 0\,, \quad \Sigma^{b}{}_{|c|ba_2\ldots a_{s-1}} = 0\,, \quad \Sigma_{\mu|b|a_1\ldots a_{s-3}c}{}^{c} = 0\,.
\end{aligned} \qquad (3.9)$$

Moreover, the parameter ξ is traceless and completely symmetric. The parameter α generalizes the Lorentz parameter for gravitation in the vielbein formalism.

In the Vasiliev formulation, the fields and gauge parameters are subject to tracelessness conditions contained in (3.1) and (3.9). It would be of interest to investigate whether these conditions can be dispensed with as in [23, 24].

3.2 Equivalence with the standard second order formulation

Since the action depends on ω only through B, extremizing it with respect to ω is equivalent to extremizing it with respect to B. Thus, we can view $\mathcal{S}^s[e,\omega]$ as $\mathcal{S}^s[e,B]$. In the action $\mathcal{S}^s[e,B]$, the field $B^{\mu\nu|a_1\ldots a_{s-1}}$ is an auxiliary field. Indeed, the field equations for $B^{\mu\nu|a_1\ldots a_{s-1}}$ enable one to express B in terms of the vielbein and its derivatives as,

$$B^{\mu\nu|a_1\ldots a_{s-1}} = 2\partial^{[\mu} e^{\nu]|a_1\ldots a_{s-1}} \qquad (3.10)$$

(the field ω is thus fixed up to the pure gauge component related to Σ.) When substituted into (3.3), (3.10) gives an action $S^s[e,B(e)]$ invariant under (3.7).

The field $e_{\mu|a_1\ldots a_{s-1}}$ can be represented by

$$e_{\mu|a_1\ldots a_{s-1}} = h_{\mu a_1\ldots a_{s-1}} + \frac{(s-1)(s-2)}{2s}\left[\eta_{\mu(a_1}h_{a_2\ldots a_{s-1})} - \eta_{(a_1a_2}h_{\mu a_3\ldots a_{s-1})}\right] + \beta_{\mu|a_1\ldots a_{s-1}}\,, \qquad (3.11)$$

where $h_{\mu a_1\ldots a_{s-1}}$ is completely symmetric, $h_{a_2\ldots a_{s-1}} = h^{\mu}{}_{\mu\ldots a_{s-1}}$ is its trace, and the component $\beta_{\mu|a_1\ldots a_{s-1}}$ possesses the symmetries of the parameter α in (3.7) and thus disappears from $S^s[e,\omega(e)]$. Of course, the double trace $h^{\mu\nu}{}_{\mu\nu\ldots a_{s-1}}$ of $h_{\mu a_1\ldots a_{s-1}}$ vanishes. The action $S^s[e(h)]$ is nothing but the one given in [17] for a completely symmetric and double-traceless bosonic spin-s gauge field $h_{\mu a_1\ldots a_{s-1}}$.

In the spin-2 case, the Vasiliev fields are $e_{\mu|a}$ and $\omega_{\nu|b|a}$ with $\omega_{\nu|b|a} = -\omega_{\nu|a|b}$. The Σ-gauge invariance is absent since the conditions $\Sigma_{\nu|b|a} = -\Sigma_{\nu|a|b}$, $\Sigma_{b|c|a} = \Sigma_{c|b|a}$ imply $\Sigma_{\nu|a|b} = 0$. The gauge transformations read

$$\delta e_{\nu|a} = \partial_\nu \xi_a + \alpha_{\nu|a}\,, \qquad \delta\omega_{\nu|b|a} = \partial_\nu \alpha_{b|a} \tag{3.12}$$

with $\alpha_{\nu|a} = -\alpha_{a|\nu}$. The relation between ω and B is invertible and the action (3.3) is explicitly given by

$$S^2[e,B] = -2\int d^D x \left[\left(B_{a[\nu|\mu]} - \frac{1}{2}B_{\nu\mu|a}\right)\left(\partial^{[\mu}e^{\nu]|a} - \frac{1}{4}B^{\mu\nu|a}\right) + 2B^{\rho}{}_{\mu|\rho}\left(\partial^{[\mu}e^{\nu]|}{}_{\nu} - \frac{1}{4}B^{\mu\nu|}{}_{\nu}\right)\right]. \tag{3.13}$$

The coefficient $Y_{\mu\nu|a}$ of the antisymmetrized derivative $\partial^{[\mu}e^{\nu]|a}$ of the vielbein is given in terms of B by

$$Y_{\mu\nu|a} = B_{a[\mu|\nu]} - \frac{1}{2}B_{\mu\nu|a} - 2\eta_{a[\mu}B_{\nu]b|}{}^{b}\,. \tag{3.14}$$

This relation can be inverted to yield B in terms of Y,

$$B_{\mu\nu|a} = 2Y_{a[\mu|\nu]} - \frac{2}{D-2}\eta_{a[\mu}Y_{\nu]b|}{}^{b}\,. \tag{3.15}$$

Re-expressing the action in terms of $e_{\mu a}$ and $Y_{\mu\nu a}$ gives the action (2.1) considered previously.

4. Spin-3 duality

Before dealing with duality in the general spin-s case, we treat in detail the spin-3 case.

4.1 Arbitrary dimension ≥ 4

Following the spin-2 procedure, we first rewrite the action (3.3) in terms of $e_{\nu|\rho\sigma}$ and the coefficient $Y_{\mu\nu|\rho\sigma}$ of the antisymmetrized derivatives of $e_{\nu|\rho\sigma}$ in the action. In terms of $\omega_{\mu|\nu|\rho\sigma}$, this field is given by

$$Y_{\mu\nu|\rho\sigma} = 2\left[\omega_{\rho|[\nu|\mu]\sigma} + \omega_{\sigma|[\nu|\mu]\rho} - 2\omega^{\lambda}{}_{|[\lambda|\mu](\rho}\eta_{\sigma)\nu} + 2\omega^{\lambda}{}_{|[\lambda|\nu](\rho}\eta_{\sigma)\mu}\right] \tag{4.1}$$

or, equivalently

$$Y_{\mu\nu|a_1a_2} = B_{a_1\mu|\nu a_2} - \frac{1}{4}B_{\mu\nu|a_1a_2} + 2\eta_{\mu a_1}B^{\lambda}{}_{\nu|\lambda a_2} + \eta_{\mu a_1}B^{\lambda}{}_{a_2|\lambda\nu} \tag{4.2}$$

where antisymmetrization in μ, ν and symmetrization in a_1, a_2 is understood. The field $Y_{\mu\nu|\rho\sigma}$fulfills the algebraic relations $Y_{\mu\nu|\rho\sigma} = Y_{[\mu\nu]|\rho\sigma} = Y_{\mu\nu|(\rho\sigma)}$ and $Y_{\mu\nu|\beta}{}^{\beta} = 0$.

One can invert (4.2) to express the field $B_{\mu\nu|\rho\sigma}$ in terms of $Y_{\mu\nu|\rho\sigma}$. One gets

$$B_{\mu\nu|\rho\sigma} = \frac{4}{3}\left[Y_{\mu\nu|\rho\sigma} + 2\left[Y_{\rho[\mu|\nu]\sigma} + Y_{\sigma[\mu|\nu]\rho}\right] + \frac{2}{D-1}\left[-2\eta_{\rho\sigma}Y_{\lambda[\mu|\nu]}{}^{\lambda} + Y^{\lambda}_{\rho|\lambda[\nu}\eta_{\mu]\sigma} + Y^{\lambda}_{\sigma|\lambda[\nu}\eta_{\mu]\rho}\right]\right]. \tag{4.3}$$

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When inserted into the action, this yields

$$\mathcal{S}(e_{\mu|\nu\rho}, Y_{\mu\nu|\rho\sigma}) = -2\int d^D x \Big\{ Y_{\mu\nu|\rho\sigma}\partial^\mu e^{\nu|\rho\sigma} + \frac{4}{3}\times \\ \times\left[\frac{1}{4}Y^{\mu\nu|\rho\sigma}Y_{\mu\nu|\rho\sigma} - Y^{\mu\nu|\rho\sigma}Y_{\rho\nu|\mu\sigma} + \frac{1}{D-1}Y^{\rho\mu|\nu}{}_\rho Y_{\lambda\nu|\mu}{}^\lambda\right]\Big\}. \tag{4.4}$$

The generalized vielbein $e_{\nu|\rho\sigma}$ may again be viewed as a Lagrange multiplier since it occurs linearly. Its equations of motion force the constraints

$$\partial^\mu Y_{\mu\nu|\rho\sigma} = 0 \tag{4.5}$$

The solution of this equation is $Y_{\mu\nu|\rho\sigma} = \partial^\lambda Y_{\lambda\mu\nu|\rho\sigma}$ where $Y_{\lambda\mu\nu|\rho\sigma} = Y_{[\lambda\mu\nu]|\rho\sigma} = Y_{\lambda\mu\nu|(\rho\sigma)}$ and $Y_{\lambda\mu\nu|\rho}{}^\rho = 0$. The action then becomes

$$\mathcal{S}(Y_{\lambda\mu\nu|\rho\sigma}) = \frac{8}{3}\int d^D x\left[-\frac{1}{4}Y^{\mu\nu|\rho\sigma}Y_{\mu\nu|\rho\sigma} + Y^{\mu\nu|\rho\sigma}Y_{\rho\nu|\mu\sigma} - \frac{1}{D-1}Y^{\rho\mu|\nu}{}_\rho Y_{\lambda\mu|\nu}{}^\lambda\right], \tag{4.6}$$

where $Y_{\mu\nu|\rho\sigma}$ must now be viewed as the dependent field $Y_{\mu\nu|\rho\sigma} = \partial^\lambda Y_{\lambda\mu\nu|\rho\sigma}$.

One now decomposes the field $Y_{\lambda\mu\nu|\rho\sigma}$ into irreducible components,

$$Y^{\lambda\nu\mu|}{}_{\rho\sigma} = X^{\lambda\nu\mu|}{}_{\rho\sigma} + \delta^{[\lambda}_{(\rho}Z^{\mu\nu]}{}_{\sigma)} \tag{4.7}$$

with $X^{\lambda\nu\mu|}{}_{\rho\mu} = 0$, $X^{\lambda\nu\mu|}{}_{\rho\sigma} = X^{[\lambda\nu\mu]|}{}_{\rho\sigma}$, $X^{\lambda\nu\mu|}{}_{\rho\sigma} = X^{\lambda\nu\mu|}{}_{(\rho\sigma)}$ and $Z^{\mu\nu}{}_\sigma = Z^{[\mu\nu]}{}_\sigma$. Since $Z^{\mu\nu}{}_\sigma$ is defined by (4.7) only up to the addition of a term like $\delta^{[\mu}_\sigma k^{\nu]}$ with k^ν arbitrary, one may assume $Z^{\mu\nu}{}_\nu = 0$. The new feature with respect to spin 2 is that the field $Z^{\mu\nu}{}_\sigma$ is now not entirely pure gauge. However, that component of $Z^{\mu\nu}{}_\sigma$ which is not pure gauge is entirely determined by $X^{\lambda\nu\mu|}{}_{\rho\sigma}$.

Indeed, the tracelessness condition $Y^{\lambda\nu\mu|}{}_{\rho\sigma}\eta^{\rho\sigma} = 0$ implies

$$Z^{[\lambda\mu|\nu]} = -X^{\lambda\nu\mu|}{}_{\rho\sigma}\eta^{\rho\sigma}\,. \tag{4.8}$$

One can further decompose $Z_{\lambda\mu|\nu} = \Phi_{\lambda\mu\nu} + \frac{4}{3}\Psi_{[\lambda|\mu]\nu}$ with $\Phi_{\lambda\mu\nu} = \Phi_{[\lambda\mu\nu]} = Z_{[\lambda\mu|\nu]}$ and $\Psi_{\lambda|\mu\nu} = \Psi_{\lambda|(\mu\nu)} = Z_{\lambda(\mu|\nu)}$. In addition, $\Psi_{(\lambda|\mu\nu)} = Z_{(\lambda\mu|\nu)} = 0$ and $\Psi_{\lambda|\mu\nu}\eta^{\mu\nu} = Z_{\lambda\mu|\nu}\eta^{\mu\nu} = 0$. Furthermore, the α-gauge symmetry reads $\delta Z_{\lambda\mu|\nu} = \alpha_{[\lambda|\mu]\nu}$ i.e, $\delta\Phi_{\lambda\mu\nu} = 0$ and $\delta\Psi_{\lambda|\mu\nu} = \frac{3}{4}\alpha_{\lambda|\mu\nu}$. Thus, the Ψ-component of Z can be gauged away while its Φ-component is fixed by X. The only remaining field in the action is $X^{\lambda\nu\mu|}{}_{\rho\sigma}$, as in the spin-2 case.

Also as in the spin-2 case, there is a redundancy in the solution of the constraint (4.5) for $Y_{\nu\alpha|\beta\gamma}$, leading to the gauge symmetry (in addition to the α-gauge symmetry)

$$\delta Y^{\lambda\mu\nu|}{}_{a_1a_2} = \partial_\rho\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2} \tag{4.9}$$

where $\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2}$ is antisymmetric in ρ, λ, μ, ν and symmetric in a_1, a_2 and is traceless on a_1, a_2, $\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2}\eta^{a_1a_2} = 0$. This gives, for X,

$$\delta X^{\lambda\mu\nu|}{}_{a_1a_2} = \partial_\rho\left(\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2} + \frac{6}{D-1}\delta^{[\lambda}_{(a_1}\psi^{\mu\nu]\rho\sigma|}{}_{a_2)\sigma}\right). \tag{4.10}$$

4.2 $D = 5$ and $D = 4$

One can then trade the field X for a field T obtained by dualizing on the indices λ, μ, ν with the ϵ-symbol. We shall carry out the computations only in the case $D = 5$ and $D = 4$, since the case of general dimensions will be covered below for general spins. Dualising in $D = 5$ gives $X^{\lambda\nu\mu|}{}_{\rho\sigma} = \frac{1}{2}\epsilon^{\lambda\nu\mu\alpha\beta}T_{\alpha\beta|\rho\sigma}$ and the action becomes:

$$\begin{aligned}\mathcal{S}(T_{\mu\nu|\rho\sigma}) = \frac{2}{3}\int d^5x\Big[&-\partial_\lambda T_{\mu\nu|\rho\sigma}\partial^\lambda T^{\mu\nu|\rho\sigma} + 2\partial^\lambda T_{\lambda\nu|\rho\sigma}\partial_\mu T^{\mu\nu|\rho\sigma} + 2\partial^\rho T_{\mu\nu|\rho\sigma}\partial_\lambda T^{\mu\nu|\lambda\sigma} + \\ &+ 8T_{\mu\nu|\rho\sigma}\partial^{\mu\rho}T_\lambda{}^{\nu|\lambda\sigma} + 2T_{\mu\nu|\rho\sigma}\partial^{\rho\sigma}T^{\mu\nu|\lambda}{}_\lambda + 4\partial_\rho T_\lambda{}^{\nu|\lambda\sigma}\partial^\rho T^\mu{}_{\nu|\mu\sigma} - \\ &- 4\partial_\nu T_\lambda{}^{\nu|\lambda\sigma}\partial^\rho T^\mu{}_{\rho|\mu\sigma} + 4\partial_\sigma T_\lambda{}^{\nu|\lambda\sigma}\partial^\rho T_{\rho\nu|\mu}{}^\mu + \partial_\lambda T^{\mu\nu|\rho}{}_\rho\partial^\lambda T_{\mu\nu|\sigma}{}^\sigma\Big]\end{aligned} \tag{4.11}$$

with $T_{\mu\nu|\rho\sigma} = T_{\mu\nu|(\rho\sigma)} = T_{[\mu\nu]|\rho\sigma}$ and $T_{[\mu\nu|\rho]\sigma} = 0$. The gauge symmetries of the T field following from (4.9) are

$$\delta T_{\mu\nu|\rho\sigma} = -\partial_{[\mu}\varphi_{\nu]|\sigma\rho} + \frac{3}{4}\left[\partial_{[\mu}\varphi_{\nu|\sigma]\rho} + \partial_{[\mu}\varphi_{\nu|\rho]\sigma}\right], \tag{4.12}$$

where the gauge parameter $\varphi_{\alpha|\rho\sigma} \sim \epsilon_{\alpha\lambda\mu\nu\tau}\psi^{\lambda\mu\nu\tau|}{}_{\rho\sigma}$ is such that $\varphi_{\alpha|\rho\sigma} = \varphi_{\alpha|(\rho\sigma)}$ and $\varphi_{\alpha|\rho}{}^\rho = 0$. The parameter $\varphi_{\alpha|\rho\sigma}$ can be decomposed into irreducible components: $\varphi_{\alpha|\rho\sigma} = \chi_{\alpha\rho\sigma} + \phi_{\alpha(\rho|\sigma)}$ where $\chi_{\alpha\rho\sigma} = \varphi_{(\alpha|\rho\sigma)}$ and $\phi_{\alpha\rho|\sigma} = \frac{3}{4}\varphi_{[\alpha|\rho]\sigma}$. The gauge transformation then reads

$$\delta T_{\mu\nu|\rho\sigma} = \partial_{[\mu}\chi_{\nu]\rho\sigma} + \frac{1}{8}\left[-2\partial_{[\mu}\phi_{\nu]\rho|\sigma} + 3\phi_{\mu\nu|(\sigma,\rho)}\right], \tag{4.13}$$

and the new gauge parameters are constrained by the condition $\chi_{\alpha|\rho}{}^\rho + \phi_{\alpha|\rho}{}^\rho = 0$.

These are the action and gauge symmetries for the field $T_{\mu\nu|\rho\sigma}$ dual to $e_{(\mu\nu\rho)}$ in $D = 5$ and coincide with the ones given in [25, 26, 27, 5].

In four spacetime dimensions, dualization reads $T_{\alpha\rho\sigma} = \epsilon_{\lambda\mu\nu\alpha}X^{\lambda\mu\nu|}{}_{\rho\sigma}$. The field $T_{\alpha\rho\sigma}$ is totally symmetric because of $X^{\lambda\nu\mu|}{}_{\rho\mu} = 0$. The action reads

$$\begin{aligned}\mathcal{S}(T_{\mu\nu\rho}) = -\frac{4}{3}\int d^4x\Big[&\partial_\lambda T_{\mu\nu\rho}\partial^\lambda T^{\mu\nu\rho} - 3\partial^\mu T_{\mu\nu\rho}\partial_\lambda T^{\lambda\nu\rho} - 6T_\lambda{}^{\lambda\mu}\partial^{\nu\rho}T_{\mu\nu\rho} - \\ &- 3\partial_\lambda T_\mu{}^{\mu\nu}\partial_\lambda T_\rho{}^{\rho\nu} - \frac{3}{2}\partial_\lambda T^{\lambda\mu}{}_\mu\partial_\nu T^{\nu\rho}{}_\rho\Big].\end{aligned} \tag{4.14}$$

The gauge parameter $\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2}$ can be rewritten as $\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2} = (-1/2)\epsilon^{\rho\lambda\mu\nu}k_{a_1a_2}$ where $k_{a_1a_2}$ is symmetric and traceless. The gauge transformations are, in terms of T, $\delta T_{\rho\sigma\alpha} = \partial_\rho k_{\sigma\alpha} + \partial_\sigma k_{\alpha\rho} + \partial_\alpha k_{\rho\sigma}$. The dualization procedure yields back the Fronsdal action and gauge symmetries [17]. Note also that the gauge-invariant curvatures of the original field $h_{\mu\nu\rho} \equiv e_{(\mu\nu\rho)}$ and of $T_{\mu\nu\rho}$, which involve now three derivatives [28, 29], are again related on-shell by an ϵ-transformation $R_{\alpha\beta\mu\nu\rho\sigma}[h] \propto \epsilon_{\alpha\beta\alpha'\beta'}\ R^{\alpha'\beta'}{}_{\mu\nu\rho\sigma}[T]$, as they should.

5. Spin-s duality

The method for dualizing the spin-s theory follows exactly the same pattern as for spins two and three:

- First, one rewrites the action in terms of e and Y (coefficient of the antisymmetrized derivatives of the generalized vielbein in the action);
- Second, one observes that e is a Lagrange multiplier for a differential constraint on Y, which can be solved explicitly in terms of a new field Y with one more index;
- Third, one decomposes this new field into irreducible components; only one component (denoted X) remains in the action; using the ϵ-symbol, this component can be replaced by the "dual field" T.
- Fourth, one derives the gauge invariances of the dual theory from the redundancy in the description of the solution of the constraint in step 2.

We now implement these steps explicitly.

5.1 Trading B for Y

The coefficient of $\partial^{[\nu}e^{\mu]|a_1\dots a_{s-1}}$ in the action is given by

$$\begin{aligned} Y_{\mu\nu|a_1\dots a_{s-1}} &= B_{a_1\mu|\nu a_2\dots a_{s-1}} - \frac{1}{2(s-1)}B_{\mu\nu|a_1\dots a_{s-1}} + 2\eta_{\mu a_1}B^{\lambda}{}_{\nu|\lambda a_2\dots a_{s-1}} + \\ &\quad +(s-2)\eta_{\mu a_1}B^{\lambda}{}_{a_2|\lambda\nu a_3\dots a_{s-1}}\,, \end{aligned} \tag{5.1}$$

where the r.h.s. of this expression must be antisymmetrized in μ,ν and symmetrized in the indices a_i. The field $Y_{\mu\nu|a_1\dots a_{s-1}}$ is antisymmetric in μ and ν, totally symmetric in its internal indices a_i and traceless on its internal indices. One can invert (5.1) to express $B_{\mu\nu|a_1\dots a_{s-1}}$ in terms of $Y_{\mu\nu|a_1\dots a_{s-1}}$. To that end, one first computes the trace of $Y_{\mu\nu|a_1\dots a_{s-1}}$. One gets

$$Y^{\lambda}{}_{\mu|\lambda a_2\cdots a_{s-1}} = \frac{D+s-4}{2(s-1)}\left(2B^{\lambda}{}_{\mu|\lambda a_2\cdots a_{s-1}} + (s-2)B^{\lambda}{}_{(a_2|a_3\cdots a_{s-1})\lambda\mu}\right) \tag{5.2}$$

$$\Leftrightarrow B^{\lambda}{}_{\mu|\lambda a_2\cdots a_{s-1}} = \frac{2(s-1)^2}{s(D+s-4)}\left(Y^{\lambda}{}_{\mu|\lambda a_2\cdots a_{s-1}} - \left(\frac{s-2}{s-1}\right)Y^{\lambda}{}_{(a_2|a_3\cdots a_{s-1})\lambda\mu}\right) \tag{5.3}$$

Using this expression, one can then easily solve (5.1) for $B_{\mu\nu|a_1\dots a_{s-1}}$,

$$\begin{aligned} B_{\mu\nu|a_1\dots a_{s-1}} &= 2\frac{(s-1)}{s}\Bigg[(s-2)Y_{\mu\nu|a_1\dots a_{s-1}} - 2(s-1)Y_{\mu a_1|\nu a_2\dots a_{s-1}} + 2\frac{(s-1)}{(D+s-4)}\times \\ &\quad\times\Big[(s-2)\eta_{a_1a_2}Y_{\mu\rho|\nu a_3\dots a_{s-1}}{}^{\rho} - (s-2)\eta_{a_1\mu}Y_{a_2\rho|\nu a_3\dots a_{s-1}}{}^{\rho} + \\ &\quad + (s-3)\eta_{a_1\mu}Y_{\nu\rho|a_2\dots a_{s-1}}{}^{\rho}\Big]\Bigg] \end{aligned} \tag{5.4}$$

where the r.h.s. must again be antisymmetrized in μ,ν and symmetrized in the indices a_i. We have checked (5.4) using FORM (symbolic manipulation program [30]).

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The action (3.3) now reads

$$
\begin{aligned}
\mathcal{S}^s = -2\int d^Dx \Big[& Y_{\mu\nu|a_1\ldots a_{s-1}}\partial^{[\nu}e^{\mu]|a_1\ldots a_{s-1}} + \frac{(s-1)^2}{s}\times \\
& \times\Big[-Y_{\mu\nu|a_1\ldots a_{s-1}}Y^{\mu a_1|\nu a_2\ldots a_{s-1}} + \frac{(s-2)}{2(s-1)}Y_{\mu\nu|a_1\ldots a_{s-1}}Y^{\mu\nu|a_1\ldots a_{s-1}} + \\
& + \frac{1}{(D+s-4)}\Big[(s-3)Y_{\mu\nu|a_2\ldots a_{s-2}}{}^{\mu}Y^{\nu\rho|a_2\ldots a_{s-2}}{}_{\rho} - \\
& - (s-2)Y_{\mu\nu|a_2\ldots a_{s-2}}{}^{\mu}Y^{a_2\rho|\nu a_3\ldots a_{s-2}}{}_{\rho}\Big]\Big]\Big] . \qquad (5.5)
\end{aligned}
$$

It is invariant under the transformations (3.7) and (3.8)

$$
\begin{aligned}
\delta e_{\nu|a_1\ldots a_{s-1}} &= \partial_\mu \xi_{a_1\ldots a_{s-1}} + \alpha_{\nu|a_1\ldots a_{s-1}} \\
\delta Y^{\mu\nu|}{}_{a_1\ldots a_{s-1}} &= 3\partial_\lambda \delta^{[\lambda}_{(a_1}\alpha^{\mu|\nu]}{}_{a_2\ldots a_{s-1})} \, .
\end{aligned} \qquad (5.6)
$$

Recall that $\alpha_{\nu|a_1\ldots a_{s-1}}$ satisfies the relations

$$
\alpha_{(\nu|a_1\ldots a_{s-1})} = 0\,, \qquad \alpha^{\nu}{}_{|\nu a_2\ldots a_{s-1}} = 0\,, \qquad \alpha_{\nu|a_1\ldots a_{s-3}b}{}^{b} = 0\,. \qquad (5.7)
$$

while $\xi_{a_1\ldots a_{s-1}}$ is completely symmetric and traceless.

5.2 Eliminating the constraint

The field equation for $e^{\mu|a_1\ldots a_{s-1}}$ is a constraint for the field Y,

$$
\partial^\nu Y_{\nu\mu|a_1\ldots a_{s-1}} = 0 \qquad (5.8)
$$

which implies:

$$
Y_{\mu\nu|a_1\ldots a_{s-1}} = \partial^\lambda Y_{\lambda\mu\nu|a_1\ldots a_{s-1}} \qquad (5.9)
$$

where $Y_{\lambda\mu\nu|a_1\ldots a_{s-1}} = Y_{[\lambda\mu\nu]|a_1\ldots a_{s-1}} = Y_{\lambda\mu\nu|(a_1\ldots a_{s-1})}$ and $Y^{\lambda\mu\nu|a}{}_{aa_3\ldots a_{s-1}} = 0$. If one substitutes the solution of the constraint inside the action, one gets

$$
\begin{aligned}
\mathcal{S}(Y_{\lambda\mu\nu|a_1\ldots a_{s-1}}) = -2\frac{(s-1)^2}{s}\int & d^Dx \times \\
\times\Big[& -Y_{\mu\nu|a_1\ldots a_{s-1}}Y^{\mu a_1|\nu a_2\ldots a_{s-1}} + \frac{(s-2)}{2(s-1)}Y_{\mu\nu|a_1\ldots a_{s-1}}Y^{\mu\nu|a_1\ldots a_{s-1}} + \\
& + \frac{1}{(D+s-4)}\Big[(s-3)Y_{\mu\nu|a_2\ldots a_{s-2}}{}^{\mu}Y^{\nu\rho|a_2\ldots a_{s-2}}{}_{\rho} \\
& - (s-2)Y_{\mu\nu|a_2\ldots a_{s-2}}{}^{\mu}Y^{a_2\rho|\nu a_3\ldots a_{s-2}}{}_{\rho}\Big]\Big] , \qquad (5.10)
\end{aligned}
$$

where $Y_{\mu\nu|a_1\ldots a_{s-1}} \equiv \partial^\lambda Y_{\lambda\mu\nu|a_1\ldots a_{s-1}}$. This action is invariant under the transformations

$$
\delta Y^{\lambda\mu\nu|}{}_{a_1\ldots a_{s-1}} = 3\,\delta^{[\lambda}_{(a_1}\alpha^{\mu|\nu]}{}_{a_2\ldots a_{s-1})}\,, \qquad (5.11)
$$

where $\alpha_{\nu|a_1\ldots a_{s-1}}$ satisfies the relations (5.7), as well as under the transformations

$$
\delta Y^{\lambda\mu\nu|}{}_{a_1\ldots a_{s-1}} = \partial_\rho \psi^{\rho\lambda\mu\nu|}{}_{a_1\ldots a_{s-1}}\,. \qquad (5.12)
$$

that follow from the redundancy of the parametrization of the solution of the constraint (5.8). The gauge parameter $\psi^{\rho\lambda\mu\nu|}{}_{a_1\ldots a_{s-1}}$ is subject to the algebraic conditions $\psi^{\rho\lambda\mu\nu|}{}_{a_1\ldots a_{s-1}} = \psi^{[\rho\lambda\mu\nu]|}{}_{a_1\ldots a_{s-1}} = \psi^{\rho\lambda\mu\nu|}{}_{(a_1\ldots a_{s-1})}$ and $\psi^{\rho\lambda\mu\nu|}{}_{a_1a_2\ldots a_{s-1}}\eta^{a_1a_2} = 0$.

5.3 Decomposing $Y_{\lambda\mu\nu|a_1\dots a_{s-1}}$ — dual action

The field $Y_{\lambda\mu\nu|a_1\dots a_{s-1}}$ can be decomposed into the following irreducible components

$$Y^{\lambda\mu\nu|}{}_{a_1\dots a_{s-1}} = X^{\lambda\mu\nu|}{}_{a_1\dots a_{s-1}} + \delta^{[\lambda}_{(a_1} Z^{\mu\nu]|}{}_{a_2\dots a_{s-1})} \tag{5.13}$$

where $X^{\lambda\mu\nu|}{}_{\lambda a_2\dots a_{s-1}} = 0$, $Z^{\mu\nu|}{}_{\mu a_3\dots a_{s-1}} = 0$. The condition $Y^{\lambda\mu\nu|a}{}_{a a_3\dots a_{s-1}} = 0$ implies

$$Z^{\mu\nu|a}{}_{a a_4\dots a_{s-1}} = 0\,, \tag{5.14}$$

$$Z^{[\mu\nu|\lambda]}{}_{a_3\dots a_{s-1}} = -\frac{(s-1)}{2} X^{\mu\nu\lambda|a}{}_{a a_3\dots a_{s-1}}\,. \tag{5.15}$$

The invariance (5.11) of the action involves only the field Z and reads

$$\begin{aligned} \delta X^{\lambda\mu\nu|}{}_{a_1\dots a_{s-1}} &= 0 \\ \delta Z_{\mu\nu|a_1\dots a_{s-2}} &= \alpha_{[\mu|\nu]a_1\dots a_{s-2}}\,. \end{aligned} \tag{5.16}$$

Next, one rewrites $Z_{\mu\nu|a_1\dots a_{s-2}}$ as

$$Z_{\mu\nu|a_1\dots a_{s-2}} = \frac{3(s-2)}{s}\Phi_{\mu\nu(a_1|a_2\dots a_{s-2})} + \frac{2(s-1)}{s}\Psi_{[\mu|\nu]a_1\dots a_{s-2}} \tag{5.17}$$

with $\Phi_{\mu\nu a_1|a_2\dots a_{s-2}} = Z_{[\mu\nu|a_1]a_2\dots a_{s-2}}$ and $\Psi_{\mu|\nu a_1\dots a_{s-2}} = Z_{\mu(\nu|a_1\dots a_{s-2})}$. So the irreducible component $\Phi_{\mu\nu a_1|a_2\dots a_{s-2}}$ of Z can be expressed in terms of X by the relation (5.15), while the other component $\Psi_{\mu|\nu a_1\dots a_{s-2}}$ is pure-gauge by virtue of the gauge symmetry (5.16), which does not affect $\Phi_{\mu\nu a_1|a_2\dots a_{s-2}}$ and reads $\delta\Psi_{\mu|\nu a_1\dots a_{s-2}} = (1/2)\alpha_{\mu|\nu a_1\dots a_{s-2}}$ (note that $\Psi_{\mu|\nu a_1\dots a_{s-2}}$ is subject to the same algebraic identities (5.7) as $\alpha_{\mu|\nu a_1\dots a_{s-2}}$). As a result, the only independent field appearing in $S(Y^{\lambda\mu\nu|}{}_{a_1\dots a_{s-1}})$ is $X^{\lambda\mu\nu|}{}_{a_1\dots a_{s-1}}$.

Performing the change of variables

$$X^{\lambda\mu\nu|}{}_{a_2\dots a_s} = \frac{1}{(D-3)!}\epsilon^{\lambda\mu\nu b_1\dots b_{D-3}} T_{b_1\dots b_{D-3}|a_2\dots a_s}\,, \tag{5.18}$$

the action for this field reads

$$\begin{aligned} \mathcal{S} = &-\frac{2(s-1)}{s(D-3)!}\int d^Dx \times \\ &\times\Big[\partial^e T^{b_1\dots b_{D-3}|a_2\dots a_s}\partial_e T_{b_1\dots b_{D-3}|a_2\dots a_s} - (D-3)\partial_e T^{eb_2\dots b_{D-3}|a_2\dots a_s}\partial^f T_{fb_2\dots b_{D-3}|a_2\dots a_s} + \\ &\quad + (s-1)\Big[-\partial_e T^{b_1\dots b_{D-3}|ea_3\dots a_s}\partial^f T_{b_1\dots b_{D-3}|fa_3\dots a_s} - \\ &\qquad - 2(D-3)T_g{}^{b_2\dots b_{D-3}|ga_3\dots a_s}\partial^{ef}T_{eb_2\dots b_{D-3}|fa_3\dots a_s} - \\ &\qquad - (s-2)T^{b_1\dots b_{D-3}|c}{}_c{}^{a_4\dots a_s}\partial^{ef}T_{b_1\dots b_{D-3}|efa_4\dots a_s} - \\ &\qquad - (D-3)\partial^e T_g{}^{b_2\dots b_{D-3}|ga_3\dots a_s}\partial_e T^f{}_{b_2\dots b_{D-3}|fa_3\dots a_s} - \\ &\qquad - \frac{1}{2}(s-2)\partial^e T^{b_1\dots b_{D-3}|c}{}_c{}^{a_4\dots a_s}\partial_e T_{b_1\dots b_{D-3}|d}{}^d{}_{a_4\dots a_s} + \\ &\qquad + (D-3)(D-4)\partial_e T_g{}^{eb_3\dots b_{D-3}|ga_3\dots a_s}\partial^h T^f{}_{hb_3\dots b_{D-3}|fa_3\dots a_s} - \end{aligned}$$

$$
\begin{aligned}
&- (s-2)(D-3)\partial_e T_g{}^{b_2\dots b_{D-3}|gea_4\dots a_s}\partial^f T_{fb_2\dots b_{D-3}|c}{}^{c}{}_{a_4\dots a_s} + \\
&+ \frac{1}{4}(s-2)(D-3)\partial_e T^{eb_2\dots b_{D-3}|c}{}_{c}{}^{a_4\dots a_s}\partial^f T_{fb_2\dots b_{D-3}|d}{}^{d}{}_{a_4\dots a_s} - \\
&- \frac{1}{4}(s-2)(s-3)\partial_e T^{b_1\dots b_{D-3}|c}{}_{c}{}^{ea_5\dots a_s}\partial^f T_{b_1\dots b_{D-3}|d}{}^{d}{}_{fa_5\dots a_s}\Big]\Big] . \qquad (5.19)
\end{aligned}
$$

The field $T_{b_1\dots b_{D-3}|a_2\dots a_s}$ fulfills the following algebraic properties,

$$
\begin{aligned}
T_{b_1\dots b_{D-3}|a_2\dots a_s} &= T_{[b_1\dots b_{D-3}]|a_2\dots a_s} && (5.20)\\
T_{b_1\dots b_{D-3}|a_2\dots a_s} &= T_{b_1\dots b_{D-3}|(a_2\dots a_s)} && (5.21)\\
T_{[b_1\dots b_{D-3}|a_2]\dots a_s} &= 0 && (5.22)\\
T_{b_1\dots b_{D-3}|a_2a_3a_4a_5\dots a_s}\eta^{a_2a_3}\eta^{a_4a_5} &= 0 && (5.23)\\
T_{b_1\dots b_{D-3}|a_2a_3a_4\dots a_s}\eta^{b_1a_2}\eta^{a_3a_4} &= 0 && (5.24)
\end{aligned}
$$

the last two relations following from (5.15) and (5.14).

Conversely, given a tensor $T_{b_1\dots b_{D-3}|a_2\dots a_s}$ fulfilling the above algebraic conditions, one may first reconstruct $X^{\lambda\mu\nu|}{}_{a_2\dots a_s}$ such that $X^{\lambda\mu\nu|}{}_{a_2\dots a_s} = X^{[\lambda\mu\nu]|}{}_{a_2\dots a_s}$, $X^{\lambda\mu\nu|}{}_{a_2\dots a_s} = X^{\lambda\mu\nu|}{}_{(a_2\dots a_s)}$ and $X^{\lambda\mu\nu|}{}_{\nu a_3\dots a_s} = 0$. One then gets the Φ-component of $Z^{\mu\nu|}{}_{a_2\dots a_{s-1}}$ through (5.15) and finds that it is traceless thanks to the double tracelessness conditions on $T_{b_1\dots b_{D-3}|a_2\dots a_s}$.

The equations of motion for the action (5.19) are

$$
G_{b_1\dots b_{D-3}|a_2\dots a_s} = 0\,, \qquad (5.25)
$$

where

$$
\begin{aligned}
G_{b_1\dots b_{D-3}|a_2\dots a_s} = {}& F_{b_1\dots b_{D-3}|a_2\dots a_s} - \frac{(s-1)}{4}\times \\
&\times\Big[2(D-3)\eta_{b_1a_2}F^{c}{}_{b_2\dots b_{D-3}|ca_3\dots a_s} + (s-2)\eta_{a_2a_3}F_{b_1\dots b_{D-3}|c}{}^{c}{}_{a_4\dots a_s}\Big]\,,
\end{aligned}
$$

and

$$
\begin{aligned}
F_{b_1\dots b_{D-3}|a_2\dots a_s} = {}& \partial_c\partial^c T_{b_1\dots b_{D-3}|a_2\dots a_s} - \\
&-(D-3)\partial_{b_1}\partial^c T_{cb_2\dots b_{D-3}|a_2\dots a_s} - (s-1)\partial_{a_2}\partial^c T_{b_1\dots b_{D-3}|ca_3\dots a_s} + \\
&+(s-1)\Big[(D-3)\partial_{a_2b_1}T^{c}{}_{b_2\dots b_{D-3}|ca_3\dots a_s} + \frac{(s-2)}{2}\partial_{a_2a_3}T_{b_1\dots b_{D-3}|c}{}^{c}{}_{a_4\dots a_s}\Big]\,,
\end{aligned}
$$

and where the r.h.s. of both expressions has to be antisymmetrised in $b_1\dots b_{D-3}$ and symmetrised in $a_2\dots a_s$.

5.4 Gauge symmetries of dual theory

As a consequence of (5.12), (5.13) and (5.18), the dual action is invariant under the gauge transformations:

$$
\delta T_{b_1\dots b_{D-3}|a_2\dots a_s} = \partial_{[b_1}\phi_{b_2\dots b_{D-3}]|a_2\dots a_s} + \frac{(s-1)(D-2)}{(D+s-4)}\partial_f\phi_{c_1\dots c_{D-4}|ga_3\dots a_s}\delta^{[fgc_1\dots c_{D-4}]}_{[a_2b_1\dots b_{D-3}]}\,,
$$

where the r.h.s. must be symmetrized in the indices a_i and where the gauge parameter $\phi_{b_1 \dots b_{D-4}|a_2 \dots a_s} \sim \epsilon_{b_1 \dots b_{D-4}\rho\lambda\mu\nu}\psi^{\rho\lambda\mu\nu|}{}_{a_2 \dots a_s}$ is such that $\phi_{b_1 \dots b_{D-4}|a_2 \dots a_s} = \phi_{[b_1 \dots b_{D-4}]|a_2 \dots a_s} = \phi_{b_1 \dots b_{D-4}|(a_2 \dots a_s)}$, and $\phi_{b_1 \dots b_{D-4}|}{}^{a}{}_{aa_4 \dots a_s} = 0$.

This completes the dualization procedure and provides the dual description, in terms of the field $T_{b_1 \dots b_{D-3}|a_2 \dots a_s}$, of the spin-s theory in D spacetime dimensions. Note that in four dimensions, the field $T_{b_1|a_2 \dots a_s}$ has s indices, is totally symmetric and is subject to the double tracelessness condition. One gets back in that case the original Fronsdal action, equations of motion and gauge symmetries.

6. Comments on interactions

We have investigated so far duality only at the level of the free theories. It is well known that duality becomes far more trickier in the presence of interactions. The point is that consistent, local interactions for one of the children theories may not be local for the other. For instance, in the case of p-form gauge theories, Chern-Simons terms are in that class since they involve "bare" potentials. An exception where the same interaction is local on both sides is given by the Freedman-Townsend model [31] in four dimensions, where duality relates a scalar theory (namely, non-linear σ-model) to an interacting 2-form theory.

It is interesting to analyse the difficulties at the level of the parent action. We consider the definite case of spin-2. The second-order action $\mathcal{S}[e_{ab}]$ (Eqn (2.2)) can of course be consistently deformed, leading to the Einstein action. One can extend this deformation to the action (2.1) where the auxiliary fields are included (see e.g. [10]). In fact, auxiliary fields are never obstructions since they do not contribute to the local BRST cohomology [21, 32]. The problem is that one cannot go any more to the other single-field theory action $\mathcal{S}[Y]$. The interacting parent action has only one child. The reason why one cannot get rid of the vielbein field $e_{a\mu}$ is that it is no longer a Lagrange multiplier. The equations of motion for $e_{a\mu}$ are not constraints on Y, which one could solve to get an interacting, local theory on the Y-side (the possibility of doing so is in fact prevented by the no-go theorem of [33]). Rather, they mix both e and Y. Thus, one is prevented from "going down" to $\mathcal{S}[Y]$. At the same time, the other parent action corresponding to (2.5) does not exist once interactions are switched on. By contrast, in the Freedman-Townsend model, the Lagrange multiplier remains a Lagrange multiplier.

7. Conclusions

In this paper, we have analyzed duality for massless gauge theories with spin ≥ 2. We have shown how to dualize such theories, replacing the original description in terms of a totally symmetric tensor with s indices by a dual description involving a tensor with mixed Young symmetry type characterized by one columns with $D-3$ boxes and $s-1$ columns with one box. Our results encompass previous analyses where duality was studied at the level of the curvatures and equations of motion, but not at the level of the action.

A crucial role is played in the approach by the first-order formulation due to Vasiliev [18], which provides the "parent action" connecting the two dual formulations (up to minor redefinitions). First-order formulations associated with exotic tensor gauge fields were considered recently in [34]. As a by-product of the analysis, we reproduce the known local actions leading to second-order field equations for "exotic" tensor gauge fields transforming in the representation of the linear group characterized by a Young diagram with one column with k boxes and m columns with one box and subject to double-tracelessness conditions.

We have considered here original gauge theories described by totally symmetric gauge fields only. It would be of interest to extend the construction to more general tensor gauge fields. This problem is currently under investigation.

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